Potential development geometric thinking for children through Tri Uan

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ABSTRACT: *Geometric thinking is considered to be one of the most controversial thinkings. Many researches have observed that children have various opportunities to cultivate the elements of geometric thinking at different levels throughout mathematical activities. Having experimented, we found out that these puzzle games are both attractive and potential to develop visualization, imagination and thinking in terms of shapes. Tri Uan is a seven-piece tangram, which is popular in Vietnam. This article presents the research of geometric thinking components and geometric activities through Tri Uan in order to develop geometric thinking at each level in children.*

KEYWORDS: **Tri Uan; geometric thinking; observant mind; imaginary mind.**

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1. Introduction

According to Nguyen Manh Tuan [1], geometry is the study about the properties of space object (the characteristics of size: length, width, height, ...) and space relation (the position of the object is indicated by the concepts in the direction: front - after, top - bottom, left - right, distance: near - far, etc.). When learning geometry, children are not only using general thinking tasks such as analysis, synthesis, concretization, abstraction, generalization, analogy, … but also conducts other intellectual activities such as rotate the image, move the image, flip the image, zoom in or out in the brain, etc. to solve the problem. These activities are generally called geometric thinking. The question is: “Is there a tool that is appropriate for measuring and developing a child’s mathematical thinking?” In the study of the Tri Uan - a Tangram composed of seven puzzle pieces, we found that perpendicular, parallel, equilateral and symmetric relations appear on this games. We have researched, tested, and realized the potential for developing geometric thinking in children that can be exploited from the Tri Uan through engaging activities.

2. Research content

2.1. Geometric thinking

In the world, there are many researches about the types of thinking that arise in geometric activities, with different names such as geometric thinking, spatial thinking, spatial intelligence, visual thinking, spatial ability.

According to L. L. Thurstone [15], spatial thinking is a component of visual thinking. Spatial thinking is represented by three elements S_1, S_2 and S_3

S_1: The ability to recognize the identity of an object when it is seen from different angles.

S_2: The ability to imagine the movement or internal displacement among the parts of a configuration that one is thinking about.

S_3: The ability to think about those spatial relations in which the body orientation of the observer is an essential part of the problem.

The assessment activities about spatial thinking of humans in L. L. Thurstone’s research is performed by a “visible sample”. Therefore, the research will cause limitations when experiments on the blind. Although spatial intelligence can even develop in an individual who is blind and therefore has no direct access to the visual world [5].

Howard Gardner showed that central to spatial intelligence are capacities to perceive the visual world accurately, to perform transformations and modifications upon one’s initial perceptions, and to be able to recreate aspects of one’s visual experience, even in the absence or relevant physical stimulate. He introduces the components of spatial intelligence including:

i) The ability to recognize instances of the same element;

ii) The ability to transform or to recognize a transformation of one element into another;

iii) The capacity to conjure up mental imagery and then to transform that imagery;

iv) The capacity to produce a graphic likeness of spatial information; and the like.

In addition, perceptual ability refers to properties such as the invariance of perception: recognizing that some of the characteristics of objects are independent of material, color and location in space. Spatial intelligence and other spatial thinking studies (for example: L. L. Thurstone's study) have the same characteristics as the difference in name and scope of the study in which the author intended.

According to Van Hieles [8], geometric thinking is divided into five levels (5 levels):

Level 1 (visual level): At this level, the child's perception of space is what surrounds them; geometry is viewed as the whole rather than the components, the characteristics constituting them (number edge or length of the edge, measure of angles). Children assess shapes based on perceptual objects, not by logical inference.

Level 2 (analysis level): Appearing of analyzing shape in children. Through their experience in practical and educational activities, children are beginning to recognize the properties of geometry, which are the basis for the classification of geometry.

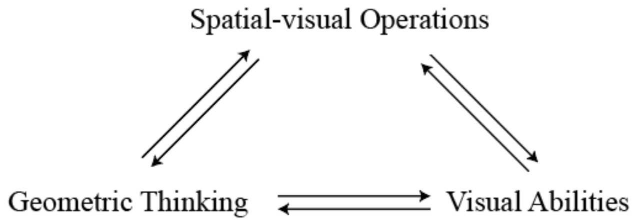
Level 3 (informal level): Children can make correct judgments about the relationship between geometry, expressing the necessary and sufficient conditions for a square, rectangle, etc. However, the statements are not present in the form of hypothetical logic - conclusions.

Level 4 (logical level): Children can accurately determine the truth value of a proposition about the relationship between shapes or the properties of shapes and the inverse proposition in geometry.

Level 5 (abstract level): Children have ability to perceive the geometric axiom plays a decisive role in the abstracted form of the axiomatic method, with complete abstraction of the nature of objects and relations are discussed in the axiomatization theory.

At level 1, children tend to develop spatial intelligence rather than logical thinking, at which point the child has not learned the exact structure of concepts as well as the axiomatic approach in geometry. Children can perform the rotation, moving picture, find the hidden face of the picture, ... to transform the space symbol. At higher levels, children develop more logical thinking when discovering the characteristics of geometry.

According to Pinkernell [13], spatial abilities are allocated in three categories, spatial-visual operations, geometric thinking and visual abilities. Therefore, spatial-visual operations include the ability to mentally construct or reproduce objects, to transform their spatial-visual characteristics. Geometric thinking includes abilities such as recognizing and understanding spatial objects, describing them and their transformation under consideration of their geometrical characteristics. Finally, visual abilities are the interpretation and construction of different representation forms of spatial-visual objects, such as models, graphs and the description of configurations using the correct terms for the underlying concepts.



Spatial ability model, Pinkernell

Angle Mizze commented that geometric thinking in Pinkernell's spatial ability model playing a major role for the application and fostering of spatial abilities in mathematics classrooms [11].

In our opinion, the following elements needed to be research deepen in order to develop the geometric thinking of children:

F1. The ability to observe and reproduce the pattern based on geometric characteristics;

F2. The ability to spatially orientate and imagine the substitution or displacement of the part of the pattern.

What activities contribute to the development of geometric thinking?

According to P.Ia.Galperin, manipulating with objects or with materials, which means manipulating with its varieties such as drawings, diagrams, models, patterns ... of real objects, is the source of all intellectual action completed. The purpose is to analyze, to separate the true content of the psychic act in the material object (or materialization). The content of this step is that the subject uses these hand to deploy actions, practice, generalize and shorten it. Thus, the action itself with objects, external action is the basis for subsequent manipulation in the children [12].

According to Piaget and Inhelder [14], children "can only "abstract" the idea of such a relation as equality on the basis of an action of equalization, the idea of a straight line from the action of following by hand or eye without changing direction, and the idea of an angle from two intersecting movements". Specifically, children's ideas about shapes do not come from passive looking. Instead, they come as children's bodies, hands, eyes...and minds... engage in action. In addition, the experiment illustrates that children need to explore shapes extensively to fully understand them. [2].

From the research on Clements [1] and Greabell [6], we found out the enormous benefits of using a variety of manipulating activities to help children learning about space and geometry since then develop their geometric thinking.

According to our research, developing the geometric thinking in children required to manipulate real objects, the real context through engaging learning activities.

2.2. Tri Uan

Tri Uan is one kind of Tangram which named following its creator Nguyen Tri Uan (1916-1995), a well-known Vietnamese in the 40s of the 20th century. This game consists of 7 pieces cut from a rectangle which size 72mmx90mm to create: a pentagon, two equal triangles, and four square trapeziums. There are two equal small trapezoids, two larger and different squares of the same square (see Figure 1).

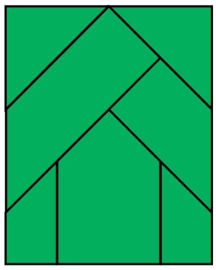


Figure 1: The model of the Uranus toy

This game can be made by paper, cardboard, wooden. Later players created a variety of shapes from seven

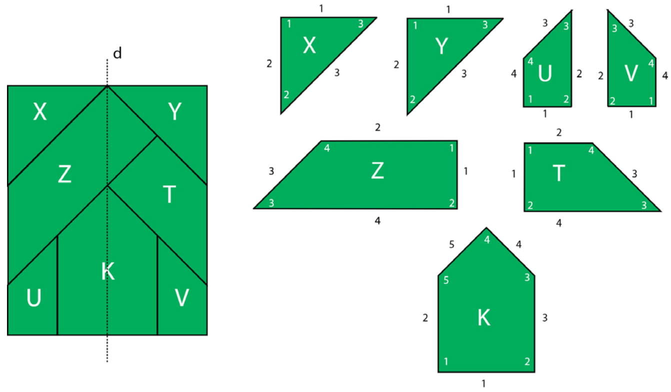


Figure 2: Figure the number and angle of each puzzle in the Aggregate Game

pieces of wood, with themes of people, labor, buildings, animals, plants to letters or numbers. According to incomplete statistics, about 1500 such figures have been created, and this number does not stop there, but is increasing day by day [17, 18, 19].

In our research, the Tri Uan is exploited and used to organize geometric activities. We named seven pieces of the Tri Uan as X, Y, Z, T, U, K, and V, and numbered the sides and angles of each piece as shown in Figure 2.

Table 1: Table describing relations in Tri Uan

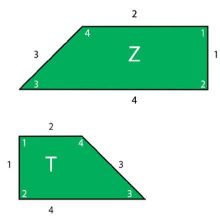
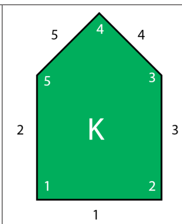
Illustrated pieces			
Equal relations	**Equal relations of side**	$SZ_1 = ST_1; SZ_3 = ST_3$	$SK_2 = SK_3; SK_4 = SK_5$
	Equal relations of angle	$AZ_1 = AZ_2 = AT_1 = AT_2$ $AZ_3 = AT_3; AZ_4 = AT_4$	$AK_1 = AK_2; AK_4 = AK_5$
Parallel relations		$SZ_1 \parallel SZ_4; ST_2 \parallel ST_4; SK_2 \parallel SK_3$	
Perpendicular relations		$SZ_1 \perp SZ_2; ST_1 \perp SZ_4; ST_1 \perp ST_2; ST_1 \perp ST_4;$ $SK_1 \perp SK_2; SK_1 \perp SK_3$	
Symmetric relations		In Tri Uan: + X and Y are symmetric through axis d. + U and V are symmetric through axis d.	d is the symmetric axis of K.

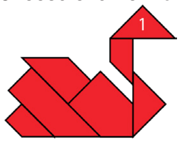
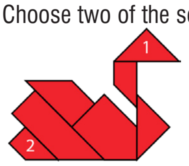
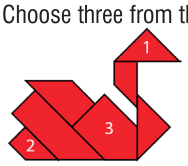
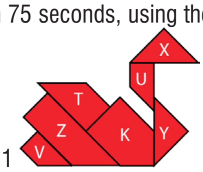
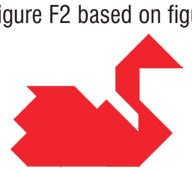
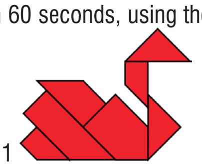
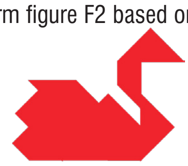
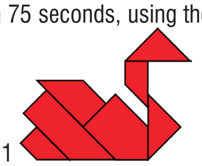
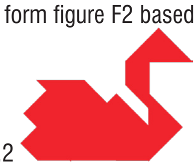
Table 2: Quality criteria of behavior indicators elements competency F1; F2

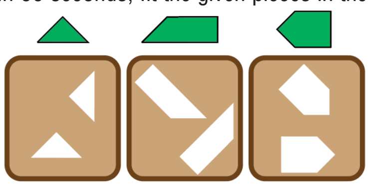
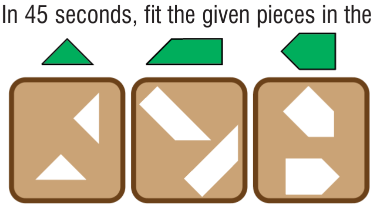
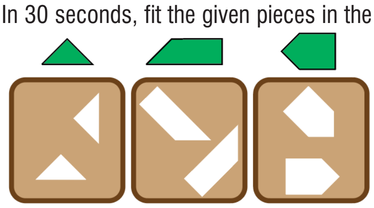
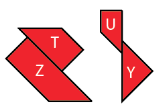
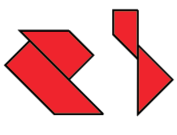
Elements	Behavior indicators	Quality criteria
F1. The ability to observe and reproduce the pattern based on geometric characteristics	1. Observe and select pieces based on geometric characteristics.	1. Choose some pieces to fit into the pattern without any geometric characteristics (edges, angles, dimensions, etc.).
		2. Choose some pieces to fit into the pattern based on one geometric characteristic (edges, angles, dimensions, etc.).
		3. Choose some pieces to fit into the pattern based on up to two geometric characteristics (edges, angles, dimensions, etc.).

Elements	Behavior indicators	Quality criteria
	2. Observe and select pieces in the given time.	1. For a given time, remember and position of three pieces correctly in the pattern
		2. For a given time, remember and position of 5 pieces correctly in the pattern
		3. For a given time, remember and position of 7 pieces correctly in the pattern
	3. Create a new one, name it and explain the reasons.	1. Use three pieces to create a new one, name the picture, and explain the reasons.
		2. Use 5 pieces to create a new one, name the picture, and explain the reasons.
		3. Use 7 pieces to create a new one, name the picture, and explain the reasons.
F2. The ability to spatially orientate and imagine the substitution or displacement of the part of the pattern.	1. Determine the direction of the puzzle	1. Fit a given piece at corresponding shapes in two different directions.
		2. Fit two given pieces at corresponding shapes in two different directions.
		3. Fit three given pieces at corresponding shapes in two different directions.
	2. Locate the missing pieces in the picture	1. Placing the 7th piece is missing that has related to other pieces in the pattern.
		2. Placing the 6th and 7th piece are missing that have related to other pieces in the pattern.
		3. Placing the 5th, 6th and 7th piece are missing that have related to other pieces in the pattern.

Table 3 illustrates the geometric activities of each level

Table 3: Geometric activities through Tri Uan at each level

Behavior indicators	Levels	Activities
1. Observe and select pieces based on geometric characteristics.	1	Choose one from the seven pieces of Tri Uan for the location (1), then explain your reasons. 
	2, 3	Choose two of the seven pieces of Tri Uan for the location (1) and (2), then explain your reasons. 
	4, 5	Choose three from the seven pieces of Tri Uan for the location (1), (2) and (3), then explain your reasons 
2. Observe and select pieces in the given time.	1	In 75 seconds, using the Tri Uan to form figure F2 based on figure F1. F.1  F.2 
	2, 3	In 60 seconds, using the Tri Uan to form figure F2 based on figure F1. F.1  F.2 
	4, 5	In 75 seconds, using the Tri Uan to form figure F2 based on figure F1. F.1  F.2 

Behavior indicators	Levels	Activities
3. Create a new one, name it and explain the reasons.	1	Use some pieces from Tri Uan to simulate the following figure and explain the reasons. 
	2, 3	Use some pieces from Tri Uan to simulate the following figure and explain the reasons.  
	4, 5	Use some pieces from Tri Uan to simulate a familiar object and explain the reasons.
1. Determine the direction of the puzzle	1	In 60 seconds, fit the given pieces in the board, respectively 
	2, 3	In 45 seconds, fit the given pieces in the board, respectively 
	4, 5	In 30 seconds, fit the given pieces in the board, respectively 
2. Locate the missing pieces in the picture	1	Placing the missing pieces in the correct place in figure F.2 to form figure F.1  F.1  F.2
	2, 3	Placing the missing pieces in the correct place in figure F.2 to form figure F.1  F.1  F.2
	4, 5	Placing the missing pieces in the correct place in figure F.2 to form figure F.1  F.1  F.2

Denote: SK_1 is the first side of the piece K; AK_1 is the angle 1 of the peice K, d is the symmetry axis of Tri Uan.

The following table describes the equality relationship, the perpendicular relation, the parallel relation appearing Tri Uan:

2.3. Developing geometric thinking through the Tri Uan

We have applied this game to help students:

+ Performing the operation: observe, imagine.

+ Installation, grafting, picture creation and getting familiar with the basic shapes and their geometric properties.

+ Developing geometric thinking.

To assess the performance for each student, we present table 2.3.1, which shows the quality criteria for each behavior on a scale of three levels (one being the lowest and three being highest).

3. Conclusion and discussion

Developing geometric thinking for students is a goal in mathematics education in Vietnam. Our research is based on the desire to design activities that use simple material linked to children's routines. Having experimented, we found out that Tri Uan is both attractive and potential to develop visualization, imagination and thinking in terms of shapes. At the same time, we initially assess the development of geometric thinking in children.

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